

RG

# Secondary Flow (circulation)

8/16/2019

①



5-20% of the magnitude of vel. of primary flow  $A$

- 1) Curved channel (e.g. meandering river)
- 2) Ekman boundary layers
- 3) Differential Advection

But the stronger  $\partial S / \partial y$  makes them of primary importance.

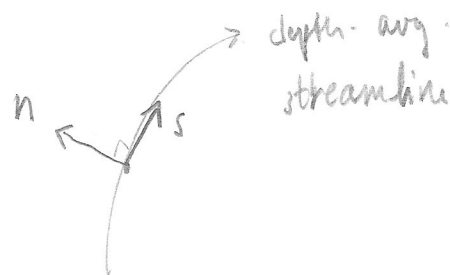
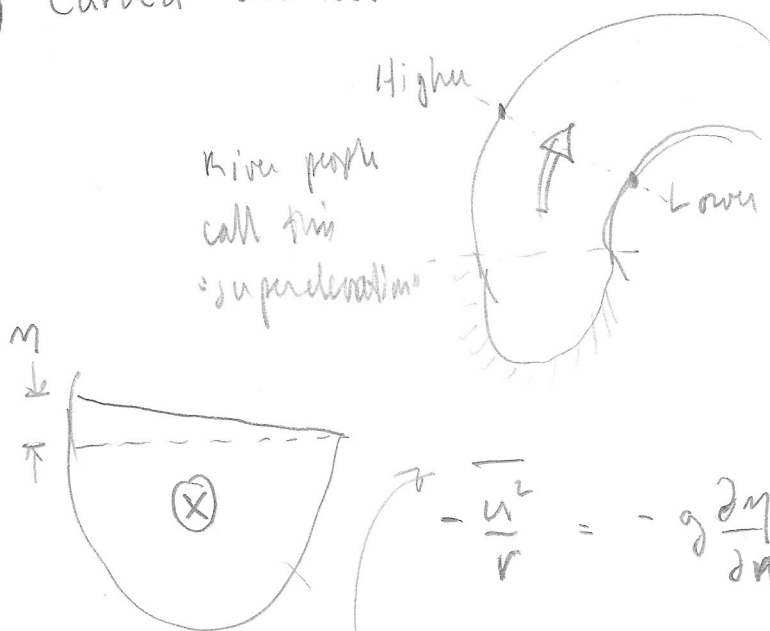


this is 'v' but it is not what we call secondary circulation

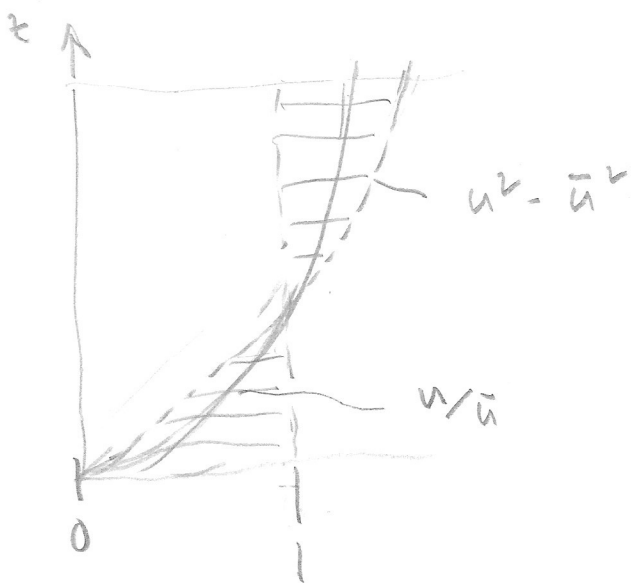
analysis approach:

rotate at each point so that  $\bar{v} = 0$  (depth avg.)

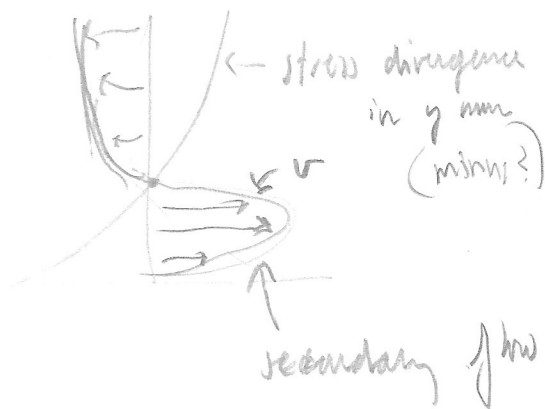
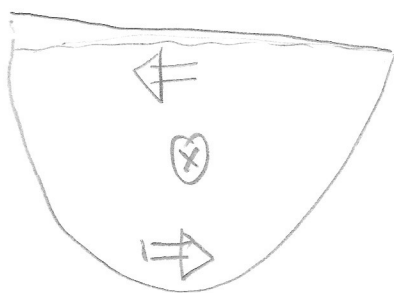
# 1) Curved channel



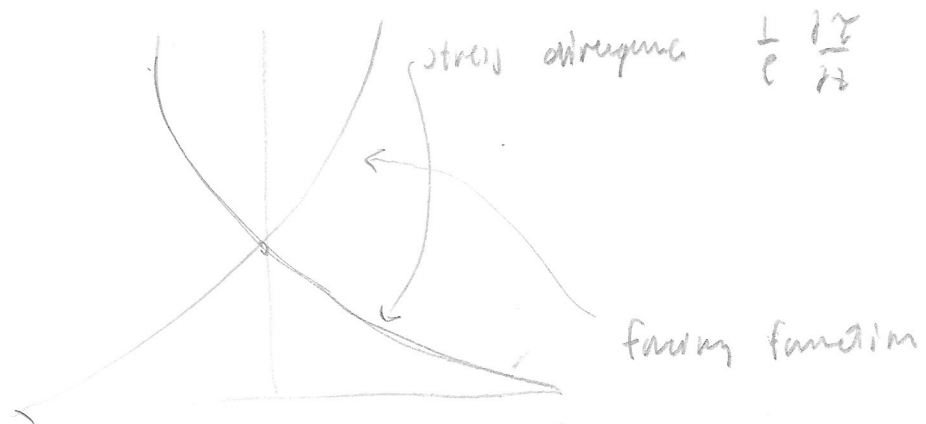
the fact that this is depth averaged is important



$\nabla$  = out of balance part of PGs driving secondary flow



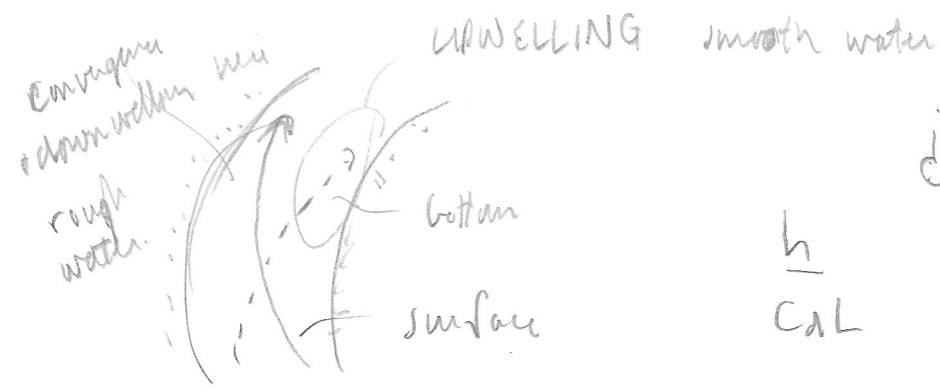
$$\frac{u^2}{r} = -K \frac{\partial u}{\partial z}$$



$$-\frac{u^2}{r} = -g\eta_n + \frac{1}{\rho} \frac{\partial \sigma}{\partial z}$$

$$-\frac{u^2}{r} = -K v_{zz}$$

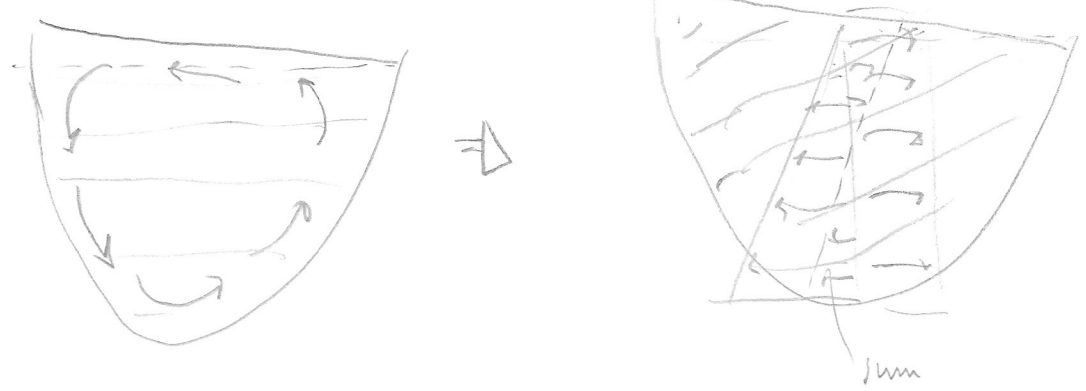
$\Rightarrow -\left(\frac{u^2}{r} - \frac{u^2}{r}\right) = -K v_{zz}$  to within a sign error.

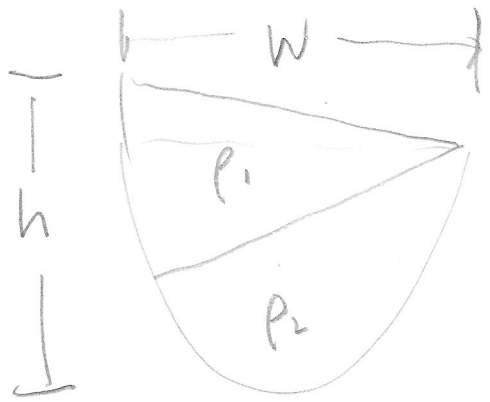


$$\frac{h}{cAL} \frac{dP}{dx}$$

advection  
 friction

what if it is stratified?





Scaling from  
Haverly sem

$$g' = \frac{g(\rho_2 - \rho_1)}{\rho_0}$$

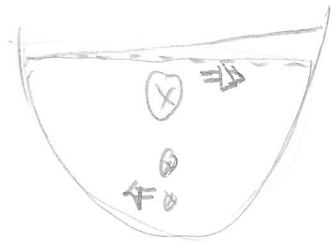
$$\frac{g'h}{W} > \frac{\frac{1}{2}(\Delta u)^2}{r}$$

or else flow overturns

$$\sim \frac{g'h}{(\Delta u)^2} > \frac{1}{2} \frac{W}{R} \leftarrow \text{same as } r$$

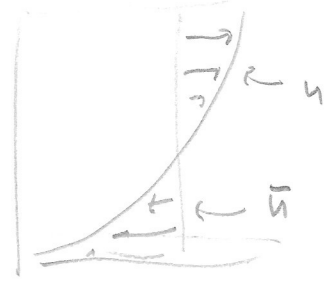
like a Richardson #, typically  $Ri > 1/4$

2) Coriolis - straight channel

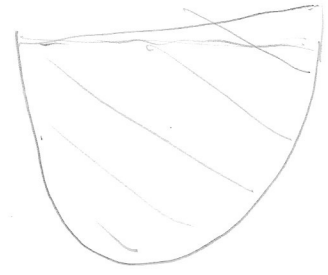


$$\bar{u} = -g \frac{\partial \eta}{\partial y}$$

Ageostrophic flow  
≡ going downhill



w/ stratification

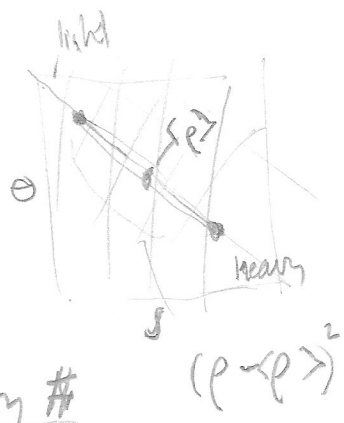


Corollis: the signs of secondary flow change flood vs. ebb.

Curvature: signs do not change in flood vs. ebb.

Scaling of the shut-down condition

$$\frac{g'h}{W} > f \Delta u \quad \approx \quad \frac{g'h}{\Delta u} = \frac{\Delta u}{fW} > 1$$



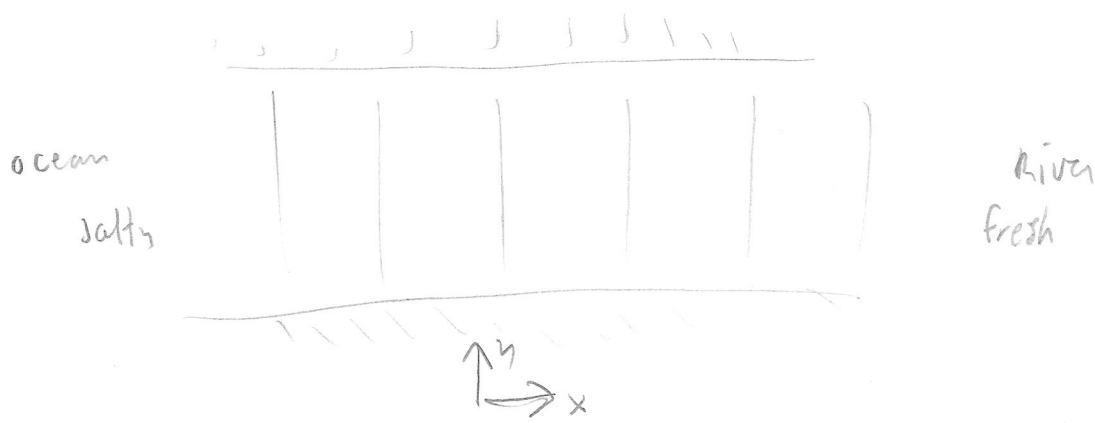
Narrow = stiff

Wide = not stiff

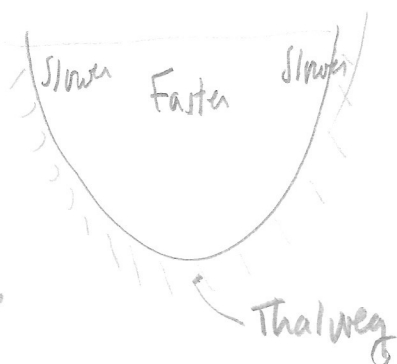
Richardson #  
a dimensionless expression of how stratified it is

Rossby #  
a dimensionless expression of how wide the system is

### 3) Differential advection

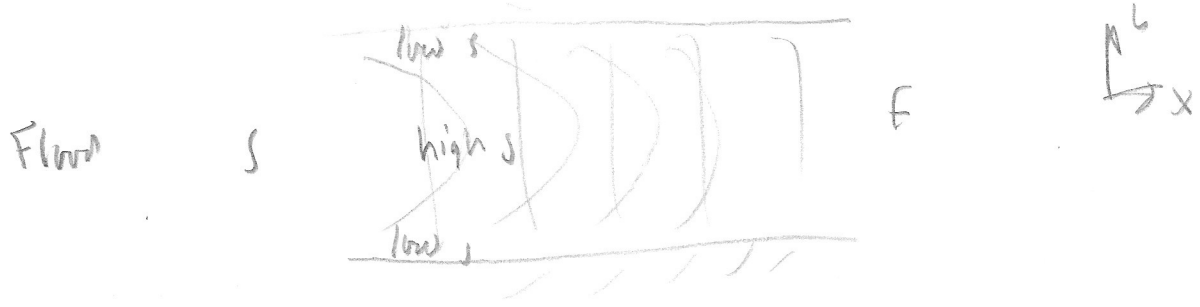


$$u \frac{\partial}{\partial x} + g \frac{\partial \eta}{\partial x} + \frac{C_d |u| u}{h} = 0$$

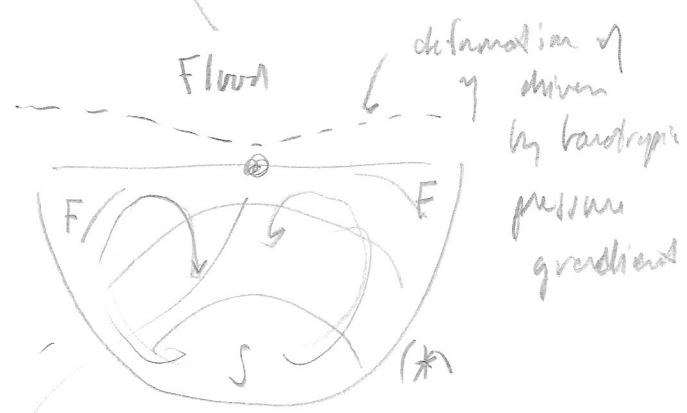


expression of how friction in shallow places

this is the main balance in shallow places



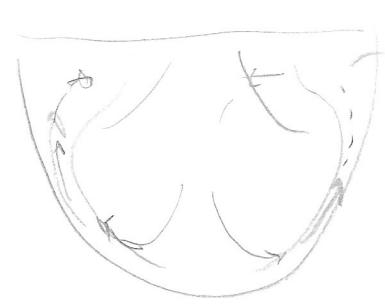
Ebb



Nunes & Simpson 1985

makes a "convergence front" down the middle of the estuary

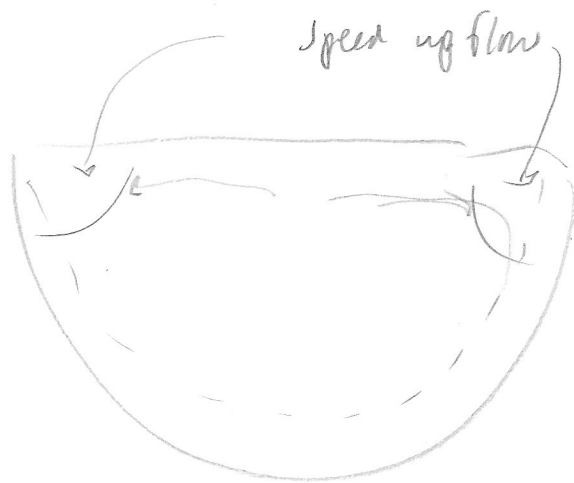
Next - from Lerczak & Geyer 2004. the structure in (\*) has an effect on the primary flow



slow flow advected up here  
FLOOD  
and \* reduces vertical shear

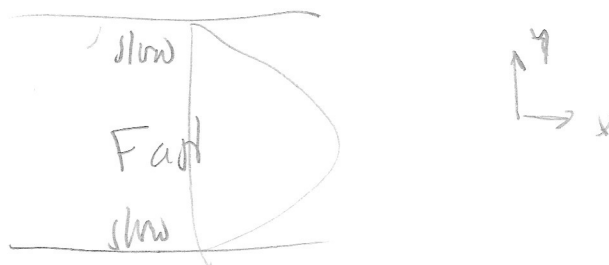
EBB

adds more vertical shear to primary flow



Both work to flatten the isopycnals

this argument stacked with cross channel shear:



Gives rise to the estuarine circulation

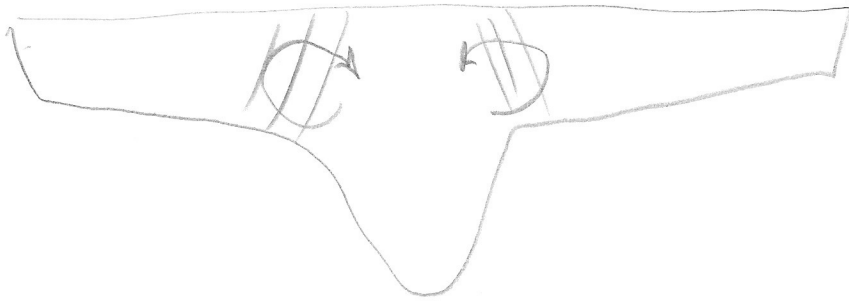


up to 1/2 augmentation of gravitational circulation

esp. true in cases w/ weak stratification, in narrow estuaries

# Delaware Bay Cobs

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concept

